The Dichotomy between Production and Consumption Decisions and Economic Efficiency

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Abstract

This paper examines possible changes in some central economic propositions due to the dichotomy between production decision and consumption decision. It finds that with economies of specialization and transaction costs, this dichotomy may imply sub-optimal decisions. Meanwhile, even without the dichotomy, the existence of economies of specialization and transaction costs makes the conventional Pareto optimal conditions not hold in general cases.

Keywords : economies of specialization, transaction costs, Pareto optimal conditions.

JEL classification codes : D00, D60, C60, B41.

1 Introduction

Classical economists like Young (1928) regarded demand and supply as two sides of the level of division of labour, which means production decisions are strongly related to consumption decisions. However, in the conventional mathematical treatment of economic topics, production decisions and consumption decisions are formalised separately. The dichotomy between production decisions and consumption decisions may not be all that important under constant returns to scale production technology or in the absence of transaction costs. However, there is a long tradition in economics, dating back to Smith (1776), that emphasizes the importance of increasing returns. And furthermore, transaction costs are observable everywhere in our daily lives. Therefore, it is important to understand how the dichotomy affects central propositions concerning economic efficiency when there are economies of specialization and transaction costs.

The classical notion, that the extent of the division of labor determines demand, supply, productivity and trade dependencies, is not usually formalised in conventional textbooks. This can be attributed to technical difficulty. The choices among what goods to purchase, to self-provide, or to sell involve corner solutions and inframarginal analysis[1]. Fortunately, we have more sophisticated tools now, which can deal with corner solutions. For example, Yang and Ng (1993) develop a microeconomic framework of consumer-producers, featuring economies of specialisation and transaction costs, and apply general equilibrium analysis to model classical propositions. In this paper, I adopt this framework to examine the efficiency implications of the traditional dichotomy between consumption and production decisions.

One of the main issues to be addressed is whether the dichotomy excludes some possible decisions which we might take and whether this leads to a suboptimal solution. The second important issue concerns possible changes in central economic propositions such as Pareto optimality conditions. In the conventional framework, the first-order conditions of Pareto optimality lead to the equality of the marginal rate of substitution of one good for another among individuals. Ng (1975) shows that the general possibility of formulating the Pareto conditions rests on the assumption that there is at least one pure private good consumed by all individuals.

[1]See Wen (1997).

He further points out that without that assumption, we have either to use interpersonal comparison of utility to achieve Pareto optimality or to face Pareto conditions much more complicated. He calls this the "Paradox of Universal Externality." In this paper, it is shown that even if all goods consumed by consumer-producers are common pure private goods, the usual form of Pareto optimality condition may not hold in the general case.

This paper is organised as follows: Section 2 describes the model and a theorem which largely reduces the technical difficulty of general equillibrium analysis in Yang-Ng's framwork. Although the theorem was proven in Wen (1997), it is extended to nonsmooth functional form in this paper. Section 3 derives the propositions relating to efficiency in such an economy. A conclusion follows in section 4.

2 The Model

In this section, we first introduce a single individual decision problem in the general framework. Then establish the theorem, which facilitates general equilibrium analysis in Yang-Ng framework and also makes our further examination on the efficiency implication of the conventional dichomoty possible.

2.1 An Individual Decision

Having abandoned the dichotomy between production decisions and consumption decisions, we assume each individual is a consumer-producer. Labor is the only initial endowment to everyone and the only variable input in the production of each good. There are m goods (for simplicity, it is assumed that they are final goods), of which the production function can be different. Labor input in producing good i is denoted as l_i. The individual consumption of good i is denoted as X_i. The product of each individual can either be consumed or sold in the market. The consumption of each good can either be self-provided or bought from the market. Denote the amount of self-provision of good i as x_i, the individual selling amount of good i as x_i^s, and the individual purchasing amount of good i as x_i^d. We further assume:

Assumption i: Utility function $U\left(X_1, X_2, \cdots, X_m\right)$ is a monotone increasing function of $X_i, \quad i = 1, 2, \cdots, m$.

Assumption ii: Production function of good i, $f_i(\cdot)$, is monotonically increasing and strictly convex. It also satisfies $f_i(0) = 0$.

Assumption iii: Transaction technology of the good i market is such that transaction cost function $c_i(\cdot)$ is concave and satisfies $0 < c_i\left(x_i^{d1} + x_i^{d2}\right) - c_i\left(x_i^{d1}\right) < x_i^{d2}, \forall x_i^{d1} \geq 0$ and $x_i^{d2} > 0$. (Or equivalently, the transaction efficiency function $k_i(\cdot)$ is convex and satisfies $0 < k_i\left(x_i^{d1} + x_i^{d2}\right) - k_i\left(x_i^{d1}\right) < x_i^{d2}, \forall x_i^{d1} \geq 0$ and $x_i^{d2} > 0$).

Assumption iv: Each consumer-producer is a price taker. If a market for good i exists, it is under perfect competition. The price of good i in terms of good 1 is denoted as p_i ($p_1 \equiv 1$).

Under the above assumptions, we have $x_i + x_i^s = f_i(l_i)$ and $X_i = x_i + k_i\left(x_i^d\right)$. Define $I = \{1, 2, \cdots, m\}$, $J = \{i \in I : x_i > 0\}$, $T = \{i \in I : x_i^s > 0\}$, $R = \{i \in I : x_i^d > 0\}$. It will be shown that $R \cap T = \emptyset$, $R \cap J = \emptyset$ and T contains at most one element. For simplicity, we assume each consumer-producer is endowed with one unit of labor. Then each individual has the constraints

$$x_i + x_i^s = f_i(l_i), \quad i \in I \tag{1}$$

$$\sum_{i \in I} p_i x_i^d \leq \sum_{i \in I} p_i x_i^s, \tag{2}$$

which yields

$$\sum_{i \in I} p_i \left(x_i + x_i^d\right) \leq \sum_{i \in I} p_i f_i(l_i). \tag{3}$$

His labor constraint is

$$\sum_{i \in I} l_i = 1. \tag{4}$$

(3) and (4) yield

$$\sum_{i=1}^{m} p_i \left(x_i + x_i^d\right) \leq \sum_{i=1}^{m-1} p_i f_i(l_i) + p_m f_m (1 - l_1 - l_2 - \cdots - l_{m-1}). \tag{5}$$

Let L_i denote the individual's labor productivity of good i $\left(L_i = \frac{f_i(l_i)}{l_i}\right)$. We say there exist economies of specialization in the production of good i if the individual's labor productivity of good i increases with the labor share in producing the good, i.e. if $\dfrac{dL_i}{dl_i} > 0$. There are economies of specialization in the production of all goods under assumption *ii*.

The consumer-producer's decision is to choose x_i, x_i^d, and l_i $(i \in I)$ to maximize utility under the budget and labor constraints, which is given by the following nonlinear programming problem:

$$\begin{aligned} \max_{x_i, x_i^d, l_i} \quad & U\left(x_1 + k_1\left(x_1^d\right), x_2 + k_2\left(x_2^d\right), \cdots, x_m + k_m\left(x_m^d\right)\right) \\ \text{s.t.} \quad & \sum_{i=1}^{m} p_i\left(x_i + x_i^d\right) \le \sum_{i=1}^{m-1} p_i f_i\left(l_i\right) + p_m f_m\left(1 - l_1 - l_2 - \cdots - l_{m-1}\right) \\ & 0 \le x_i \le f_i\left(l_i\right), \quad x_i^d \ge 0, \quad l_i \ge 0, \quad i \in I \end{aligned} \tag{6}$$

Since $l_1, l_2, \cdots, l_m$ are not included in the objective function, we need the following theorem to endogenize them in the model. Although Wen (1997) gives a proof of the theorem, it is proven under assumptions of smooth functional forms.

Theorem 1: For problem (6), the optimal decision of the individual does not involve buying and selling the same good, does not involve self-providing and buying the same good, and does not involve selling more than one good.

The proof is provided in the appendix 1.

2.2 Further General Equilibrium Analysis

The above theorem makes the computation of general equilibrium based on inframarginal analysis possible. From this theorem, we know that optimal solution of the problem (6) is always a corner solution. The agent has to compare his or her utility levels of those corner solutions that are compatible with the theorem. Autarky (a profile of decision variables with $x_i > 0$, $x_i^d = x_i^s = 0$, $\forall i \in I$) is one possible solution. If the person is in autarky, the problem (6) becomes

$$\begin{aligned} \max_{x_i} \quad & U\left(x_1, x_2, \cdots, x_m\right) = \max_{l_i} U\left(f_1\left(l_1\right), f_2\left(l_2\right), \cdots, f_m\left(l_m\right)\right) \\ \text{s.t.} \quad & l_i > 0 (i \in I), \quad \sum_{i \in I} l_i = 1 \end{aligned}$$

After the labor share in producing each good has been solved, the utility level can be calculated. If the individual is not in autarky, then he or she sells and only sells one good according to theorem 1. We denote the good the individual sells as good i_0 ($T = \{i_0\}$) without loss of generality. Suppose the agent self-provides $m-n$ goods besides the one he or she sells, so J contains $m-n+1$ ($2 \le n \le m$) elements and R contains $n-1$ elements. In these cases, the problem (6) becomes

$$\begin{aligned}
\max \quad & U\left(X_1, X_2, \cdots, X_m\right) \\
\text{s.t.} \quad & \sum_{i \in R} p_i x_i^d \le p_{i_0}\left(f_{i_0}\left(1 - \sum_{i \in J-T} l_i\right) - x_{i_0}\right) \\
& 0 < \sum_{i \in J-T} l_i < 1 \\
& x_{i_0} > 0\,, \quad l_i > 0 \quad (i \in J-T)\,, \quad x_i^d > 0 \quad (i \in R) \\
\text{where} \quad & X_i = x_i = f_i\left(l_i\right), i \in I; \quad X_{i_0} = x_{i_0}; \quad X_i = k_i\left(x_i^d\right), i \in R
\end{aligned}$$

Solving this problem for all possible i_0 and n yields the optimal labor share in producing each good in each case: l_i^* $(i \in J-T)$ and $l_{i_0}^* = 1 - \sum_{i \in J-T} l_i^*$, optimal self-provided amount of the good he or she sells $x_{i_0}^*$, optimal trade plan x_i^{d*} $(i \in R)$ and $x_{i_0}^{s*} = f_{i_0}\left(l_{i_0}^*\right) - x_{i_0}^*$, optimal consumption bundle $(X_1^*, X_2^*, \cdots, X_m^*)$, as well as the corresponding utility level $U\left(X_1^*, X_2^*, \cdots, X_m^*\right)$. Since $1 \le m-n \le m-1$, the above decision problem includes $C_m^1\left(C_{m-1}^1 + C_{m-1}^2 + \cdots + C_{m-1}^{m-1}\right) = m(2^{m-1} - 1)$ profiles of variables. Each of these profiles and the case of autarky represents the individual's different specialization levels in producing each of the m goods. Comparing the utility levels across all these profiles, the individual can identify the optimal production plan, the optimal trade plan, and the optimal consumption plan. The optimal decisions yield the maximum of utility which is dependent on the individual's utility function, production functions, relative prices of goods, and transaction efficiency of each good.

Each feasible combination of agents in different configurations will constitute a social pattern of division of labor.[2] Autarky is a special feasible case where every consumer-producer self-provides each and every good he consumes. Under the assumption that there is free entry to each production (or say, there is freedom of professional choices), an equalibrium pattern[3] of social division of labor will generate utility equalization among agents in the different configurations in the pattern. But

[2]If there can be relative prices which equalize total market demand for each good and total supply of the good in a combination, we say the combination is feasible.

[3]An equilibrium combination not only requires the existence of relative prices to clear markets, but also require the utility equalization among agents in different configurations.

the general equalibrium pattern of social division of labor is the one which generates the highest utility to each agent. Multiple general equilibria are possible[4]

3 The Results Concerning Efficiency

3.1 The Efficiency Implication of the Dichotomy

This section uses a simple example to establish the following proposition 1 and show the consequences of the simplified treatment by separating production decisions and consumption decisions.

Proposition 1 The separation between production decision and consumption decision of each consumer-producer can lead to non-optimal decisions when transaction costs incurred in some goods market outweigh the economies of specialization.

Example: $m = 3$, $U = XYZ$; $f_x(l_x) = l_x^2$, $f_y(l_y) = l_y^3$, $f_z(l_z) = l_z^2$; $p_x \equiv 1$, $p_y = 0.5$, $p_z = 2$; $k_x\left(x^d\right) = \frac{1}{20}x^d$, $k_y\left(y^d\right) = \frac{4}{5}y^d$, $k_z\left(z^d\right) = \frac{1}{2}z^d$; $l_x + l_y + l_z = 1$. This example satisfies assumption $i - -iv$. Since the consumer-producer has no other initial endowment and does not sell labor directly, he or she will make production decision first under neoclassical dichotomy between consumption decision and production decision. As total cost of the agent's production is one unit of labor, so total profit maximization is equivalent to total revenue maximization:

$$\max_{l_x, l_z}\left(l_x^2 + 2l_z^2 + 0.5\left(1 - l_x - l_z\right)^3\right)$$

The solution of this problem is $l_x = 0$, $l_y = 0$, $l_z = 1$, i.e. the consumer-producer will totally specialize in producing good z and choose the configuration of selling good z but buying both goods x and y. Then as a consumer in the neoclassical framework, the individual chooses trade plan to maximize his or her utility as follows:

$$\begin{aligned} \max_{x^d, y^d, z} \quad & U = \left(\frac{1}{20}x^d\right) z \left(\frac{4}{5}y^d\right) \\ \text{s.t.} \quad & x^d + 0.5y^d \leq 2(1 - z) \\ & 0 < z < 1\,, \quad x^d > 0\,, \quad y^d > 0 \end{aligned}$$

[4] Existence of general equilibrium is proven in Lin et al. (1998).

The solution is $x^d = \frac{2}{3}$, $y^d = \frac{4}{3}$, $z = \frac{1}{3}$, and $U^0 = 0.0119$.

However, in the configuration of self-providing good x, selling good z and buying good y, the maximization problem is

$$\begin{aligned} \max_{l_x, y^d, z} \quad & U = l_x^2 \left(\frac{4}{5} y^d\right) z \\ \text{s.t.} \quad & 0.5 y^d \leq 2 \left((1 - l_x)^2 - z\right) \\ & 0 < l_x < 1\,, \quad y^d > 0\,, \quad z > 0 \end{aligned}$$

The solution of this problem is $l_x = \frac{1}{3}, \quad y^d = \frac{8}{9}, \quad z = \frac{2}{9}$, and $U = 0.0176$.

Obviously, the consumer-producer can reach higher utility level in the configuration of self-providing good x, selling good z and buying good y than totally specializing in producing good z. Furthermore, the optimal production plan, optimal trade plan, optimal consumption plan as well as the maximal utility level comes from the configuration which leads to the highest utility level, while the configuration of totally specializing in producing good z and the configuration of self-providing good x, selling good z and buying good y are only two possible candidates. In other words, the maximum utility level U^* that the consumer-producer can reach is higher than or equal to 0.0176. Therefore, we must have $U^* > U^0$.

¿From the above example, we can see that the dichotomy between consumption decision and production decision can lead to non-optimal decisions when the transaction costs in some good markets outweigh the economies of specialization. Hence, the dichotomy is a theoretical limitation as it excludes possible individual choices. In fact, whenever trade incurs transaction cost, the dichotomy between production and consumption can lead to suboptimal decisions. Appendix A.2 gives out a proof when production technology is constant return to scale. Similar to what Coase (1988) emphasized, ignoring transaction costs make conventional economics lack a theory of the firm and be incapable of addressing why market exists. The dichotomy between production decision and consumption decision shifts the central economic topic from classical concern for the relationship among individual level of specialization, social level of division of labor, a nation's productivity and people's well-being to resource allocations.

3.2 Changes of Efficiency Conditions in the Presence of Economies of Specialization and Transactions Costs

To show possible central proposition changes, we establish following two propositions.

Proposition 2 At the optimal decision, the last dollar a consumer-producer spends on each traded good does not bring him the same marginal utility, generally speaking.

Proof: Without loss of generality, we assume the optimal configuration is the one the agent sells good 1, self-provides goods $1, 2, \cdots, m-n+1$ and buys goods $m-n+2, m-n+3, \cdots, m$. This indicates the following maximal problem has a solution which gives him or her the highest utility level among all possible configurations.

$$\begin{aligned} \max \quad & U = U\left(x_1, f_2\left(l_2\right), \cdots, f_{m-n+1}\left(l_{m-n+1}\right), k_{m-n+2}\left(x^d_{m-n+2}\right), \cdots, k_m\left(x^d_m\right)\right) \\ \text{s.t.} \quad & \sum_{m-n+2}^{m} p_k x^d_k \leq p_1\left(f_1\left(1-\sum_{2}^{m-n+1} l_i\right) - x_1\right) \\ & x_1 > 0; \quad l_i > 0\,, i = 2, 3, \cdots, m-n+1; \quad x^d_r > 0\,, r = m-n+2, \cdots, m \end{aligned}$$

For simplicity, we look at the case that $f_j\left(\cdot\right)$ and $k_j\left(\cdot\right)$ are differentiable, $j = 1, 2, \cdots, m$. From the first order conditions, we have

$$\frac{U_1}{U_r} = \frac{p_1}{p_r/k'_r\left(x^d_r\right)} \quad \text{and} \quad \frac{U_j}{U_r} = \frac{p_j/k'_j\left(x^d_j\right)}{p_r/k'_r\left(x^d_r\right)}\,, \quad j, r \in [m-n+2, m]$$

That is

$$\frac{dU}{dp_1 x_1} = k'_r\left(x^d_r\right)\frac{dU}{dp_r x^d_r} \quad \text{and} \quad \frac{dU}{dp_j x^d_j} = \frac{k'_r\left(x^d_r\right)}{k'_j\left(x^d_j\right)}\frac{dU}{dp_r x^d_r}\,, \quad j, r \in [m-n+2, m]$$

Because $0 < k'_r = 1 - c'_r < 1$ and usually $k'_r\left(x^d_r\right) \neq k'_j\left(x^d_j\right)$, hence, $\frac{dU}{dp_1 x_1} \neq \frac{dU}{dp_r x^d_r}$ and $\frac{dU}{dp_j x^d_j} \neq \frac{dU}{dp_r x^d_r}$, i.e. the last dollar spent on each traded good does not bring the same marginal utility. Q.E.D.

When transaction cost is taken into account, the last dollar an agent spends on each traded good usually does not bring the same marginal utility in terms of markets prices. It worths noting that we cannot simply change the price to an "effective price" by dividing the market price with a unit transaction efficiency to make the equality hold because the marginal transaction efficiency in trading a good depends on the individual's endogenous trading plan. In conventional framework, the interdependency between the optimal individual level of specialization and individual demand and supply for each good is simply ignored.

Proposition 3 Optimal resource allocation in general equilibrium usually does not lead to the equalization of the marginal rate of substitution between traded goods among individuals.

Proof: For simplicity, we assume all individuals are ex ante identical. Otherwise, we have to use supercripts to distinguish the utility function, the production functions and the transactions technologies of different agents, which does not change the proof but only complicates the notations. For any two traded goods j and r, one of following three cases must be true in general equilibrium structure:

(1)there are two consumer-producers: one sells good j and also buys good r, and the other sells good r and also buys good j. Like in the proof of last proposition, we look at the case that $f_j(\cdot)$ and $k_j(\cdot)$ are differentiable, $j = 1, 2, \cdots, m$. In this case, from the first order conditions of the configurations of person 1 and 2, we can get $\frac{U_j}{U_r}|_1 = \frac{p_j}{p_r/k_r'(x_r^d)}$ and $\frac{U_j}{U_r}|_2 = \frac{p_j/k_j'\left(x_j^d\right)}{p_r}$, where $\frac{U_j}{U_r}|_1$ and $\frac{U_j}{U_r}|_2$ are the marginal rate of substitution for good j with good r of person 1 and person 2, respectively. Since $0 < k_j', k_r' < 1$, $\frac{U_j}{U_r}|_1 \neq \frac{U_j}{U_r}|_2$.

(2)the consumer-producer who sells good j buys good r, but the consumer-producer who sells good r self-provides good j. In this case, the first order condition gives us $\frac{U_j}{U_r}|_1 = \frac{p_j}{p_r/k_r'(x_r^d)}$, but $\frac{U_j}{U_r}|_2 = \frac{1/f_j'\left(l_j^2\right)}{1/f_r'(l_r^2)}$, where $\frac{U_j}{U_r}|_1$ is the marginal rate of substitution for good j with good r of person who sells good j and buys good r while $\frac{U_j}{U_r}|_2$ is the marginal rate of substitution for good j with good r of person who sells good r but self-provides good j, l_j^2 and l_r^2 are the labor share the latter inputs in the production of goods j and r respectively. Hence, it is usual that $\frac{U_j}{U_r}|_1 \neq \frac{U_j}{U_r}|_2$.

(3)the consumer-producer who sells good j self-provides good r, and the consumer-producer who sells good r self-provides good j. In this case, $\frac{U_j}{U_r}|_1 = \frac{1/f_j'\left(l_j^1\right)}{1/f_r'(l_r^1)}$ and $\frac{U_j}{U_r}|_2 = \frac{1/f_j'\left(l_j^2\right)}{1/f_r'(l_r^2)}$. Since we assume ex ante identical individuals, for these two types of consumer-producer to get same utility level in two different configurations (one is to sell good j and the other is to sell good r, there must be $l_j^1 > l_j^2$ and $l_r^1 < l_r^2$.

And usually, $\frac{U_j}{U_r}|_1 \neq \frac{U_j}{U_r}|_2$. Q.E.D.

¿From the proof of proposition 3, the conventional first order conditions for Pareto optimality do not hold in the presence of economies of specilization and transaction costs. Instead, the first order conditions are related to individual level of specilization, social pattern of division of labor, and individuals' endogenous trade plan.

4 Conclusion and Summary

In this paper, it is shown that in the presence of transaction costs, conventional dichotomy between production and consumption decisions usually leads an individual to non-optimal decisions no matter whether there is economies of specialization. In presence of transaction costs, profit maximization can lead individual to productions which he/she has a comparative advantage. This may make the individual pay high transaction costs in consumption good which he/she doesn't produce, thus lead the individual to a low utility level. In the presence of economies of specilization, the dichotomy will lead individual to total specialization. Hence, the dichotomy is a theoretical shortcoming as (1) it ignores the tradeoff between the gain from comparative advantage in production and the transaction costs in trades; (2)when there is economies of specialization, it excludes individual's self-provision of some consumption goods. This discovery seems especially important to Labour Economics, Trade Theory and Welfare Economics. In Labour Economics, individual labour supply is a key topic. From the example illustrating the proposition 1 and the example in Appendix A.2, we see the allocation of individual labor force with the dichotomy will be very different from the optimal allocation of individual labor force without the dichotomy. In trade, transaction costs are important factors for a country to consider in which productions it should specialize. But with the dichotomy, specialization simply according to comparative advantage in production can lead a country to a low utility level[5].

[5]For further studies, see Cheng et al. (1999). The proposition 7 of the paper shows that if price movements are caused by changes in transaction conditions, the Stolper-Samuelson theorem may not hold outside or within the diversification cone. The Rybczynski theorem and factor equilization theorem do not hold if there are transaction costs for international trade.

In fact, the dichotomy shifts the central topic from the relationship between the organisational changes based on evolution in division of labor and national prosperity to resource allocation problems. In addition, we have shown that the conventional Pareto optimal conditions do not hold when there are economies of specialization and transaction costs even without the dichtomy. As the first order conditions are usally related to individual level of specilization, social pattern of division of labor, and individuals' endogenous trade plan, application of conventional Pareto optimal conditions and Edgeworth box analysis in reality may cause inefficient allocation.

Appendix

A.1 Proof of Theorem 1

In this appendix, for problem (6), we prove the theorem 1. To prove the theorem 1, we need establish following claims.

Claim 1: An individual does not buy and sell the same good, i.e. $R \cap T = \emptyset$.

Proof: Suppose not. Let $x_i^0, x_i^{d0}, l_i^0 (i \in I)$ denote the solution of the problem (6). Then $\exists j \in T \cap R$, such that $x_j^{d0} > 0$, $x_j^{s0} = f_j\left(l_j^0\right) - x_j^0 > 0$, from which there is either $x_j^{s0} > x_j^{d0} > 0$ or $x_j^{d0} \geq x_j^{s0} > 0$.

Step 1: If $x_j^{s0} > x_j^{d0} > 0$, denote $x_j^* = x_j^0 + x_j^{d0}$, $x_j^{d*} = 0$, $x_j^{s*} = x_j^{s0} - x_j^{d0} > 0$, and $x_i^* = x_i^0$, $x_i^{d*} = x_i^{d0}$, $x_i^{s*} = x_j^{s0}$ for $i \neq j$, $i \in I$. It is easy to verify that labor allocation is the same between the values of the decision variables with asterisks and the optimal ones, i.e. $l_i^* = f_i^{-1}(x_i^* + x_i^{s*}) = f_i^{-1}(x_i^0 + x_i^{s0}) = l_i^0$, $\forall i \in I$. Furthermore, $x_i^*, x_i^{d*}, l_i^* (i \in I)$ is a feasible solution of the problem (6) too. However, from assumption iii, we have $k_j\left(x_j^{d0}\right) < x_j^{d0}$. Hence, $X_j^* = x_j^* + k_j\left(x_j^{d*}\right) = x_j^0 + x_j^{d0} > X_j^0 = x_j^0 + k_j\left(x_j^{d0}\right)$ while $X_i^* = X_i^0$ for $i \neq j, i \in I$. Since U is monotonically increasing, $U\left(X_1^*, X_2^*, \cdots, X_m^*\right) > U\left(X_1^0, X_2^0, \cdots, X_m^0\right)$. This contradicts the fact that $x_i^0, x_i^{d0}, l_i^0 (i \in I)$ is the solution of the problem (6). Actually, $\forall i \in I$, if $x_j^{d0} > 0$, $x_j^{s0} > 0$ and $x_j^{d0} < x_i^{s0}$, the person can increase his utility by rearranging his trade plan of the goods without making any production adjustment. So, it is impossible that $x_i^{d0} > 0$, $x_i^{s0} > 0$ hold simultaneously if $x_i^{s0} > x_i^{d0}$ ($\forall i \in I$).

Step 2: If $x_j^{d0} \geq x_j^{s0} > 0$, denote $x_j^* = x_j^0 + x_j^{s0}$, $x_j^{d*} = x_j^{d0} - x_j^{s0} \geq 0$, $x_j^{s*} = 0$, and $x_i^* = x_i^0$, $x_i^{d*} = x_i^{d0}$, $x_i^{s*} = x_i^{s0}$ for $i \neq j$, $i \in I$. It is easy to verify that labor allocation is the same between the values of the decision variables with asterisks and of the optimal ones, i.e. $l_i^* = f_i^{-1}(x_i^* + x_i^{s*}) = f_i^{-1}(x_i^0 + x_i^{s0}) = l_i^0$, $\forall i \in I$. Furthermore, $x_i^*, x_i^{d*}, l_i^*(i \in I)$ is a feasible solution of the problem (6) too. However, from assumption iii, we have $k_j\left(x_j^{d0}\right) - k_j\left(x_j^{d0} - x_j^{s0}\right) < x_j^{s0}$. Hence, $X_j^* = x_j^* + k_j\left(x_j^{d*}\right) = x_j^0 + x_j^{s0} + k_j\left(x_j^{d0} - x_j^{s0}\right) > X_j^0 = x_j^0 + k_j\left(x_j^{d0}\right)$ while $X_i^* = X_i^0$ for $i \neq j, i \in I$. Since U is monotonic increasing, we have $U\left(X_1^*, X_2^*, \cdots, X_m^*\right) > U\left(X_1^0, X_2^0, \cdots, X_m^0\right)$. This contradicts the fact that x_i^0, x_i^{d0}, l_i^0 $(i \in I)$ is the solution of the problem (6). Actually, $\forall i \in I$, if $x_i^{d0} \geq x_i^{s0} > 0$, the person can increase his utility by rearranging his trade plan of the goods without making any production adjustment. So, it is impossible that $x_i^{d0} > 0$, $x_i^{s0} > 0$ hold simultaneously if $x_i^{d0} \geq x_i^{s0}$ $(\forall i \in I)$.

Step 3: From step 1 and step 2, we know that $x_i^{d0} > 0$, $x_i^{s0} > 0$ cannot hold simultaneously no matter $x_i^{s0} > x_i^{d0}$ or $x_i^{d0} \geq x_i^{s0}$. This establishes Claim 1.

Claim 2: An individual sells one good at most.

Proof: Suppose not. Then in the solution of the problem (6), $\exists j, k \in T$, such that $x_j^0 \geq 0$, $x_j^{s0} > 0$, $x_k^0 \geq 0$, $x_k^{s0} > 0$. Let $l_j^0 = f_j^{-1}\left(x_j^0 + x_j^{s0}\right)$ and $l_k^0 = f_k^{-1}\left(x_k^0 + x_k^{s0}\right)$, $l^0 \equiv l_j^0 + l_k^0$, $\overline{l_j} \equiv f_j^{-1}\left(x_j^0\right)$, $\overline{l_k} \equiv l^0 - \overline{l_j}$, $\overline{\overline{l_k}} \equiv f_k^{-1}\left(x_k^0\right)$, and $\overline{\overline{l_j}} \equiv l^0 - \overline{\overline{l_k}}$. It is easy to verify: $\overline{l_j} < l_j^0 < \overline{\overline{l_j}}$, $\overline{\overline{l_k}} < l_k^0 < \overline{l_k}$, and $l_j^0 + l_k^0 = \overline{l_j} + \overline{l_k} = \overline{\overline{l_j}} + \overline{\overline{l_k}} = l^0$. Let $g\left(l_j\right) = p_j f_j\left(l_j\right) + p_k f_k\left(l^0 - l_j\right)$, $\overline{l_j} \geq l_j \geq \overline{\overline{l_j}}$. As f_j, f_k are strictly convex, $g(\cdot)$ is strictly convex as well. Hence, $\max\limits_{\overline{l_j} < l_j < \overline{\overline{l_j}}} g\left(l_j\right) = \max\left(g\left(\overline{l_j}\right), g\left(\overline{\overline{l_j}}\right)\right)$, which indicates $g\left(l_j^0\right) < \max\left(g\left(\overline{l_j}\right), g\left(\overline{\overline{l_j}}\right)\right)$, i.e. $p_j(x_j^0 + x_j^{s0}) + p_k(x_k^0 + x_k^{s0}) < \max\left(p_j f_j\left(\overline{l_j}\right) + p_k f_k\left(\overline{l_k}\right), p_j f_j\left(\overline{\overline{l_j}}\right) + p_k f_k\left(\overline{\overline{l_k}}\right)\right)$. Therefore, the person can get extra receipt through rearranging his labor allocation between the production of good j and k. As U is monotonically increasing in its variables, the extra receipt can be used to buy some goods to consume, and hence to increase utility. This contradicts the assumption that x_j^0, l_j^0, x_k^0, l_k^0 is part of the solution of the problem (6). The argument by negation establishes claim 2.

Claim 3: An individual does not buy and self– provide the same good, i.e. $R \cap J = \emptyset$.

Proof: Suppose not. Then in the solution of the problem (6), $\exists j \in R \cap J$, i.e. $x_j > 0, x_j^d > 0$. T contains only one element when the individual is not in autarky according to claim 2. Denote $T = \{i_0\}$, then $x_{i_0} \geq 0$ and $x_{i_0}^s > 0$. Let $l_j \equiv f_j^{-1}(x_j)$, $l_{i_0} \equiv f_{i_0}^{-1}\left(x_{i_0} + x_{i_0}^s\right)$, $\bar{l} \equiv l_j + l_{i_0}$, $x_{i_0}^{sj} \equiv \frac{p_j}{p_{i_0}} x_j^d$, $\overline{l_{i_0}} = f_{i_0}^{-1}\left(x_{i_0} + x_{i_0}^s - x_{i_0}^{sj}\right)$. We have $0 < l_j < \overline{l_j} = \bar{l} - \overline{l_{i_0}}$. Now, let l_j varies and define $X_j(l) = x_j(l) + k_j\left(x_j^d(l)\right) = f_j(l) + k_j\left(\frac{p_{i_0}}{p_j}\left(f_{i_0}\left(\bar{l} - l\right) - f_{i_0}\left(\overline{l_{i_0}}\right)\right)\right)$, $0 \leq l \leq \overline{l_j}$. It is easy to verify that $X_j(l)$ is strictly convex. Therefore, $\max\limits_{0 \leq l \leq \overline{l_j}} X_j(l) = \max\left(X_j(0), X_j\left(\overline{l_j}\right)\right)$ and $X_j(l_j) = x_j + k_j\left(x_j^d\right) < \max\limits_{0 \leq l \leq \overline{l_j}} X_j(l)$ due to $0 < l_j < \overline{l_j}$. In otherwords, the consumption of good j can be increased through the adjustment of labor allocation between good j and the good he sells without affecting the production and consumption of the other goods. This contradicts $x_j > 0$, $x_j^d > 0$, $x_{i_0} \geq 0$, and $x_{i_0}^s > 0$ is a part of the solution of the problem (6). This argument by negation establishes claim 3.

Claims 1, 2, and 3 establish the theorem 1. Q.E.D.

A.2 The separation between production and consumption decisions of each consumer-producer can lead to non-optimal decisions when there are transaction cost even though production technology is constant return to scale.

Example: $m = 3$, $U = XYZ$; $f_x(l_x) = 2al_x$, $f_y(l_y) = 5al_y$, $f_z(l_z) = al_z$; $p_x \equiv 1$, $p_y = 0.5$, $p_z = 2$; $k_x\left(x^d\right) = \frac{1}{20}x^d$, $k_y\left(y^d\right) = \frac{4}{5}y^d$, $k_z\left(z^d\right) = \frac{2}{5}z^d$; $l_x + l_y + l_z = 1$. Since the consumer-producer has no other initial endowment and does not sell labor directly, he or she will make production decision first under neoclassical dichotomy between consumption and production decisions. As total cost of the agent's production is one unit of labor, so total profit maximization is equivalent to total revenue maximization:

$$\max_{l_x, l_y}\left(2al_x + 0.5 * 5al_y + 2 * a\left(1 - l_x - l_y\right)\right)$$

The solution of this problem is $l_x = 0$, $l_y = 1$, $l_z = 0$, i.e. the consumer-producer will totally specialize in producing good y and choose the configuration of selling good y but buying both goods x and z. Then as a consumer in the neoclassical framework, the individual chooses trade plan to maximize his or her utility as follows:

$$\max_{x^d,y,z^d} \quad U = \left(\frac{1}{20}x^d\right) y \left(\frac{2}{5}z^d\right)$$
$$\text{s.t.} \quad x^d + 2z^d \leq 0.5 * (5a - y)$$
$$0 < y < 5a\,, \quad x^d > 0\,, \quad z^d > 0$$

The solution is $x^d = \frac{5a}{6}\,, y = \frac{5a}{3}\,, z^d = \frac{5a}{12}$, and $U^0 = 0.0116a^3$.

However, in the configuration of self-providing good x, self-providing and selling good z and buying good y, the maximization problem is

$$\max_{l_x,y^d,z} \quad U = (2al_x)\left(\frac{4}{5}y^d\right) z$$
$$\text{s.t.} \quad 0.5y^d \leq 2\left(a\left(1 - l_x\right) - z\right)$$
$$0 < l_x < 1\,, \quad y^d > 0\,, \quad z > 0$$

The solution of this problem is $l_x = \frac{1}{3}\,, \quad y^d = \frac{4a}{3}\,, \quad z = \frac{a}{3}$, and $U = 0.237a^3$.

Obviously, the consumer-producer can reach higher utility level in the configuration of self-providing good x, selling good z and buying good y than totally specializing in producing good y. Furthermore, the optimal production plan, optimal trade plan, optimal consumption plan as well as the maximal utility level comes from the configuration which leads to the highest utility level, while the configuration of totally specializing in producing good z and the configuration of self-providing good x, selling good z and buying good y are only two possible candidates. In other words, the maximum utility level U^* that the consumer-producer can reach is higher than or equal to $0.237a^3$. Therefore, we must have $U^* > U^0$.

This example illustrates that the dichotomy between production and consumption decisions can lead a consumer-producer to specialize in the production where he has a comparative advantage but incur high transaction costs in buying consumption goods which he does not produce. When the transaction costs of these goods are very high, the dichotomy leads the individual to non-optimal decisions and makes him suffer utility losses. For two reasons, I put the case with transaction costs and economies of specialization in the proposition 1 but the case with transaction costs and production technology of constant return to scale in the appendix. First, the assumptions in main body of the paper are consistent. They are standard assumptions in Yang-Ng framework. Second, the existence of general equilibrium in

Yang-Ng framework with economies of specialization and transaction costs has been proven by Lin, et al. (1998). However, there is no proof on the existence of general equilibrium in an economy with constant return to scale production technology in presence of transaction costs although Kurz (1974) has proven the existence of equilibrium of an exchange economy with transaction costs.

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